Braking–Radiation: An Energy Source for a Relativistic Fireball

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Abstract

If the Schwarzschild black–hole is moving rectilinearly with uniform 3–velocity and suddenly stops, according to a distant observer, then we demonstrate that this observer will see a spherical light–like shell or "relativistic fireball" radiate outwards with energy equal to the original kinetic energy of the black–hole.

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Current models for the source of gamma ray bursts provide a strong motivation to explore, in the context of General Relativity, the construction of models of light–like spherical shells of matter ("relativistic fireball"). In the fireball model of gamma ray bursts a relativistically expanding shell is slowed down by the interstellar medium and its energy is converted to gamma rays [1]. Currently proposed sources are binary neutron star mergers [2], failed supernovae [3] and magnetic white dwarf collapse [4]. In this paper we propose a new source for a relativistic fireball. A Schwarzschild black–hole moving rectilinearly with uniform 3–velocity according to a distant observer suddenly stops. We show that this results in a spherical light–like shell or relativistic fireball propagating outwards. The total energy of the fireball measured by the distant observer is the same as the original kinetic energy of the black–hole. In our model the mechanism for the production of the shell is not considered. However the effect of the shell in producing zero recoil velocity in the black–hole aswell as the origin of the total energy of the shell is manifest.

We use the Schwarzschild line–element in the form

$$ds^2 = k^2\, r^2 \left\{\frac{d\xi^2}{1-\xi^2} + (1-\xi^2)\, d\phi^2\right\} - 2\, du\, dr - \left(1 - \frac{2\, m}{r}\right) du^2 \ , \qquad (1)$$

with $k^{-1} = \gamma\,(1 - v\,\xi)$, v is a real constant such that $0 < v < 1$ and $\gamma = (1-v^2)^{-1/2}$. We can remove the parameter v from (1) and recover the usual form of the Schwarzschild line–element by making the replacement

$$\xi \longrightarrow \frac{v+\xi}{1+v\,\xi} \ . \qquad (2)$$

The inverse of this transformation is a Lorentz boost in the direction $\xi = +1$ viewed by a distant observer [5]. In this space–time $u =$ constant are null hypersurfaces generated by the geodesic integral curves of the (future–pointing) null vector field $\partial/\partial r$, with r an affine parameter along them. These null hypersurfaces are future–directed null–cones in the sense that the generators are geodesic, shear–free and have expansion r^{-1}.

We now subdivide the space–time with line–element (1) into two halves M^- and M^+, each with boundary the future null–cone $\mathcal{N}(u = 0)$. To the past $(u < 0)$ of $\mathcal{N}$ the space–time M^- has line–element (1). To the future $(u > 0)$ of $\mathcal{N}$ the space–time M^+ has line–element

$$ds_+^2 = r_+^2 \left\{\frac{d\xi_+^2}{1-\xi_+^2} + (1-\xi_+^2)\, d\phi_+^2\right\} - 2\, du\, dr_+ - \left(1 - \frac{2\, m_+}{r_+}\right) du^2 \ . \qquad (3)$$

The space–times M^- and M^+ are attached on $\mathcal{N}$ with the matching conditions

$$\xi = \xi_+ \ , \phi = \phi_+ \ , r = r_+ k^{-1} \ , \qquad (4)$$

with

$$k^{-1} = \gamma\,(1 - v\,\xi_+)\ . \tag{5}$$

These conditions ensure that the metric on $\mathcal{N}$ induced by its embedding in M^- is the same as the metric on $\mathcal{N}$ induced by its embedding in M^+. A detailed motivation for this matching is given in [5]. The physical picture is as follows: relative to the observer using the plus coordinates a Schwarzschild black–hole moving rectilinearly with uniform 3–velocity v has its 3–velocity suddenly reduced to zero and this is followed by the emergence of a spherical–fronted light–like signal. For greater generality we have assumed that the rest–mass of the black–hole has changed from m in M^- to m_+ in M^+. We now consider the physical properties of the signal with history the future null–cone $\mathcal{N}$. To do this we use the theory of light–like signals in general relativity developed by Barrabès–Israel (BI) [6]. The BI theory enables us to calculate, if it exists, the coefficient of $\delta(u)$ in the Einstein tensor of $M^- \cup M^+$. This coefficient, if non–zero, is simply related to the surface stress–energy tensor of a light–like shell with history $\mathcal{N}$. The theory also enables us to calculate the coefficient of $\delta(u)$ in the Weyl tensor of $M^- \cup M^+$ if it exists. This allows us to determine whether or not the light–like signal with history $\mathcal{N}$ includes an impulsive gravitational wave [8]. For the details of the BI technique the reader must consult [6] and further developments are to be found in [7]. We will merely guide the reader through the present application of the theory. The local coordinate system in M^- with line–element (1) is denoted $\{x^\mu_-\} = \{\xi, \phi, r, u\}$ while the local coordinate system in M^+ with line–element (3) is denoted $\{x^\mu_+\} = \{\xi_+, \phi_+, r_+, u\}$. The equation of $\mathcal{N}$ is $u = 0$ and thus we take as normal to $\mathcal{N}$ the null vector field with components n_μ given via the 1–form $n_\mu dx^\mu_\pm = -du$. Since we wish to discover the physical properties of $\mathcal{N}$ observed by the observer using the plus coordinates, we take $\{\xi^a\} = \{\xi_+, \phi_+, r_+\}$ with $a = 1, 2, 3$ as intrinsic coordinates on $\mathcal{N}$. A set of three linearly independent tangent vector fields to $\mathcal{N}$ is $\left\{e_{(1)} = \partial/\partial\xi_+, e_{(2)} = \partial/\partial\phi_+, e_{(3)} = \partial/\partial r_+\right\}$. The components of these vectors on the plus side of $\mathcal{N}$ are $e^\mu_{(a)}|_+ = \delta^\mu_a$. The components of these vectors on the minus side of $\mathcal{N}$ are

$$e^\mu_{(a)}|_- = \frac{\partial x^\mu_-}{\partial \xi^a}\ , \tag{6}$$

with the relation between $\{x^\mu_-\}$ and $\{\xi^a\}$ given by the matching conditions (4). Hence we find that

$$\begin{aligned} e^\mu_{(1)}|_- &= (1, 0, -r_+\gamma\,v, 0)\ , && (7)\\ e^\mu_{(2)}|_- &= (0, 1, 0, 0)\ , && (8)\\ e^\mu_{(3)}|_- &= (0, 0, \gamma\,(1 - v\,\xi_+), 0)\ . && (9) \end{aligned}$$

We need a transversal on $\mathcal{N}$ consisting of a vector field on $\mathcal{N}$ which points out of $\mathcal{N}$. A convenient such (covariant) vector expressed in the coordinates $\{x_+^\mu\}$ is ${}^+N_\mu = (0, 0, 1, \frac{1}{2} - \frac{m_+}{r_+})$. Thus since $n^\mu = \delta_3^\mu$ we have ${}^+N_\mu n^\mu = +1$.We next construct the transversal on the minus side of $\mathcal{N}$ with covariant components ${}^-N_\mu$. To ensure that this is the same vector on the minus side of $\mathcal{N}$ as ${}^+N_\mu$ when viewed on the plus side we require

$$ {}^+N_\mu \, e^\mu_{(a)}|_+ = {}^-N_\mu \, e^\mu_{(a)}|_- \ , \qquad {}^+N_\mu {}^+N^\mu = {}^-N_\mu {}^-N^\mu \ . \tag{10} $$

The latter scalar product is zero as we have chosen to use a null transversal. We find that

$$ {}^-N_\mu = \left(\frac{r_+ v}{1 - v\,\xi_+}, 0, \frac{1}{\gamma\,(1 - v\,\xi_+)}, D \right) \ , \tag{11} $$

with

$$ D = \frac{v^2(1 - \xi_+^2)\,\gamma}{2(1 - v\,\xi_+)} + \frac{1}{2\gamma\,(1 - v\,\xi_+)} - \frac{m}{\gamma^2(1 - v\,\xi_+)^2 r_+} \ . \tag{12} $$

Next the transverse extrinsic curvature on the plus and minus sides of $\mathcal{N}$ is given by

$$ {}^\pm\mathcal{K}_{ab} = -{}^\pm N_\mu \left(\frac{\partial e^\mu_{(a)}|_\pm}{\partial \xi^b} + {}^\pm\Gamma^\mu_{\alpha\beta} \, e^\alpha_{(a)}|_\pm e^\beta_{(b)}|_\pm \right) \ , \tag{13} $$

where ${}^\pm\Gamma^\mu_{\alpha\beta}$ are the components of the Riemannian connection associated with the metric tensor of M^+ or M^- evaluated on $\mathcal{N}$. The key quantity we need is the jump in the transverse extrinsic curvature across $\mathcal{N}$ given by

$$ \sigma_{ab} = 2 \, \left({}^+\mathcal{K}_{ab} - {}^-\mathcal{K}_{ab} \right) \ . \tag{14} $$

This jump is independent of the choice of transversal on $\mathcal{N}$ [6]. We find that in the present application $\sigma_{ab} = 0$ except for

$$ \sigma_{11} = \frac{2}{1 - \xi_+^2} \, \left(m\,k^3 - m_+ \right) \ , \qquad \sigma_{22} = 2\,(1 - \xi_+^2) \left(m\,k^3 - m_+ \right) \ , \tag{15} $$

with k given by (5). Now σ_{ab} is extended to a 4–tensor field on $\mathcal{N}$ with components $\sigma_{\mu\nu}$ by padding–out with zeros (the only requirement on $\sigma_{\mu\nu}$ is $\sigma_{\mu\nu} \, e^\mu_{(a)}|_\pm \, e^\nu_{(b)}|_\pm = \sigma_{ab}$). With our choice of future–pointing normal to $\mathcal{N}$ and past–pointing transversal, the surface stress–energy tensor components are $-S_{\mu\nu}$ with $S_{\mu\nu}$ given by [6]

$$ 16\pi \, S_{\mu\nu} = 2\,\sigma_{(\mu}\,n_{\nu)} - \sigma\,n_\mu\,n_\nu - \sigma^\dagger g_{\mu\nu} \ , \tag{16} $$

with

$$ \sigma_\mu = \sigma_{\mu\nu}\,n^\nu \ , \qquad \sigma^\dagger = \sigma_\mu\,n^\mu \ , \qquad \sigma = g^{\mu\nu}\gamma_{\mu\nu} \ . \tag{17} $$

In the present case $\sigma_\mu = 0$ and thus $\sigma^\dagger = 0$ and the surface stress–energy tensor takes the form

$$- S_{\mu\nu} = \rho \, n_\mu \, n_\nu \ . \tag{18}$$

Hence the energy density of the light–like shell is [6]

$$\rho = \frac{\sigma}{16\pi} = \frac{1}{4\pi \, r_+^2} \left(m \, k^3 - m_+ \right) \ . \tag{19}$$

Thus the null–cone $\mathcal{N}$ is the history of a light–like shell with isotropic surface stress–energy given by (18). We note that $m \, k^3$ is the "mass aspect" of the black–hole, in the terminology of Bondi et al.[9], on the minus side of $\mathcal{N}$. A calculation of the singular δ–part of the Weyl tensor for $M^- \cup M^+$ reveals that it vanishes.*Hence there is no possibility of the light–like signal with history $\mathcal{N}$ containing an impulsive gravitational wave.* We note that ρ is a monotonically increasing function of ξ_+. Thus on the interval $-1 \le \xi_+ \le +1$, ρ is maximum at $\xi_+ = +1$ (in the direction of the motion of the black–hole) and ρ is minimum at $\xi_+ = -1$. This is as one would expect. A burst of null matter predominantly in the direction of motion is required to halt the black–hole. In this sense the model we have constructed here could be thought of as a limiting case of a Kinnersley rocket [10] [11].

By integrating (19) over the shell with area element $dA = r_+^2 \, d\xi_+ \, d\phi_+$ and with $-1 \le \xi_+ \le +1, 0 \le \phi_+ < 2\pi$ we obtain the total energy E of the shell measured by the distant observer who sees the black–hole, moving rectilinearly with 3–velocity v in the direction $\xi_+ = +1$, suddenly halted. Thus

$$E = \frac{1}{4\pi} \int_0^{2\pi} d\phi_+ \int_{-1}^{+1} \left(m \, k^3 - m_+ \right) d\xi_+ \ . \tag{20}$$

This results in

$$E = m \, \gamma - m_+ \ . \tag{21}$$

So the energy of the light–like shell is the difference in the relative masses of the black–hole before and after the emission of the light–like shell. When $v = 0$ ($\gamma = 1$) the energy of the shell is the difference in the rest–masses (naturally taking $m_+ < m$) and this is a well–known result [6]. If $v \neq 0$ and $m = m_+$ then

$$E = m \, (\gamma - 1) \ . \tag{22}$$

In this case all of the relative kinetic energy of the black–hole before being halted is converted into the relativistic shell.

If the fireball model which we have constructed were to have any relevance to the fireball model of gamma ray bursts then (assuming that the energy of the relativistically expanding shell is converted to gamma rays [1]) to produce

the expected energy $E = 10^{52}$ergs using a black–hole of 10^9 solar masses we see from (22) that $v = 1$km/sec approximately, which is one per cent of the speed of our galaxy relative to the Local Group.

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